Monthly global irradiation on tilted planes: a simple correlation method and its validation

Bernard Bourges

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MONTHLY GLOBAL IRRADIATION ON TILTED PLANES:

A SIMPLE CORRELATION METHOD AND ITS VALIDATION

Bernard BOURGES, Consultant

6, rue de l'Armor, 35760 St-Grégoire (France)

ABSTRACT

This paper presents a simple correlation for the computation of monthly global solar irradiation on tilted surfaces, facing due south. The conversion factor, R, from the horizontal plane to the tilted plane, is considered: it has been established from the Solar Radiation European Atlas data (computed by Pr. Page's algorithm). R is linearly correlated to the monthly clearness index, Kt. The correlation slope depends on the month, the latitude and the tilt angle. This set of correlations fits the European Atlas Data within 2% in summer and 5% in winter (tilt ang. <60deg), within 5 to 10% for South-vertical surfaces. Validation results with respect to measured data from 7 locations are also presented. In some cases, there is a slight systematic under-estimation by a few percent (Trappes, Carpentras, Albany); but generally the results seem to be consistent with the correlation scattering (Geneva, Ispra, Bracknell, Valentia).

KEYWORDS

Solar radiation; tilted plane; linear correlation; validation; measurements.

INTRODUCTION

This paper presents a simple method of estimating the monthly global irradiation on a tilted plane facing due South from the irradiation value on a horizontal plane. This latter value is available for a number of locations all over the world or may be easily computed by using Angstrom type correlations, based on sunshine hours. But the end user is generally interested in the value for a specific inclined surface, in order to design a solar thermal or photovoltaic system for example.

THE R-CORRELATION

Let us consider the conversion factor, R, from horizontal plane irradiation H to tilted plane irradiation Ht during a given month

$$R = Ht / H \tag{1}$$

A linear correlation between R and the clearness index Kt has been investigated in the form

$$R = (1 + \cos s)/2 + r . (1 - \cos s)/2 + b . Kt \tag{2}$$

where s is the slope of the surface, and r, the ground albedo.
This form is suggested by the usual relationship of R assuming the diffuse sky radiation D to be isotropic (Rb is the conversion factor for beam irradiation)

$$R = (1 - D/H) . Rb + D/H . (1 + \cos s)/2 + r (1 - \cos s)/2 \tag{3}$$

The additional assumption of D/H being a linear function of Kt leads to eqn 2. This also suggests that the slope of the correlation line, b, depends on the tilt angle, the month and the latitude.
Tilted plane data from the European Atlas (Palz, 1984a, 1984b) has been used. The values are computed rather than measured, using a complete algorithm (Page, 1986) which accounts for the diffuse sky radiation an-isotropy.
The correlation between R and Kt has been established for three tilt angles (30, 60 and 90 deg.) and all the months of the year. The slope of the correlation straight-line, b, is a quadratic function of the geographical latitude. This can be summarized in a set of charts (e.g. Fig. 1) giving R vs Kt for a given tilt angle and latitude. A polynomial interpolation is proposed in order to generalize the results to cover any tilt angle. Complete results have been published in an earlier paper (Bourges, 1987).
The accuracy of this method, with respect to the original European Atlas data used for its development, can be characterized by the relative standard- deviation of errors (Table 1) (i.e. S.D. of the ratio Rcomputed / Ratlas).
This standard- deviation is dependent on slope and month. The best accuracy is obtained in summer at low slopes (30 deg.) where standard-deviation is equal to 0.5%; the error increases with slope (1.5% to 2.0% for a vertical surface for the same summer month) and becomes the greatest during winter months (2% to 3% at 30 deg., 3% to 5% at 90 deg.).

Table 1 Relative Standard-Deviation of Errors (%) of the R-correlation

Month	1	2	3	4	5	6	7	8	9	10	11	12
Tilt=45°	3.5	2.3	1.9	1.0	1.0	0.8	0.7	1.0	1.2	1.6	2.9	4.0
Tilt=90°	4.9	3.2	2.4	1.8	2.4	2.3	1.9	1.5	1.8	2.4	3.9	5.5

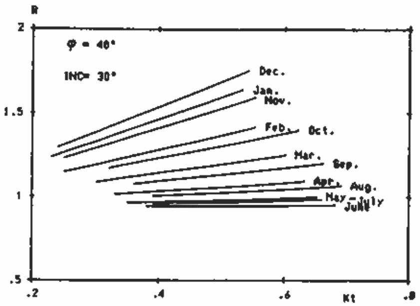

Fig. 1: Example of R-charts providing the conversion factor for tilted plane, R, as a function of the clearness index, Kt.

VALIDATION

Data used for the development of the present R-correlations consists of computed rather than measured values. Although the computation method (Page, 1986) is based on a number of experimental results, it seems to be necessary to validate the present correlation with respect to measured irradiation values on tilted planes. Such data is available for a sample of locations and for representative periods. Data used in this analysis is described in Table 2.

Table 2 Experimental Data used for the present Validation
Albedo values of 0.2 are estimated.

Location	Latit.	Country	TiltAng.	Period	Days	Alb.	Source
Albany	42.7°N	U.S.A.	43-90°	1980-81	500	0.0	A.S.R.C. (1983)
Carpentras	44.1°N	France	45-90°	1979-81	766	0.0	Dir. de la Météo.
Ispra	45.8°N	Italy	45-90°	1980-85	2000	0.2	Bollini (1986)
Geneva	46.2°N	Switz.	45-90°	1979-82	1095	0.2	Ineichen (1983)
Trappes	48.8°N	France	45-90°	1979-81	500	0.0	Dir. de la Météo.
Bracknell	51.4°N	U.K.	90°	1966-75	3000	0.0	U.K. Met. Office
Valentia	51.9°N	Irel.	90°	1976-78	730	0.2	Irish Met. Serv.

RESULTS

The results of the comparison between measured irradiation values and computed values using the former R-correlations are presented in Table 3 and fig.2. They have to be analyzed with respect to the scattering of the original R-correlations, as summarized in Table 1, keeping in mind that most of the values (95%) must be in the range ± 2.5 S.D. (Confidence interval in the gaussian assumption).
Results are quite satisfactory and consistent with this assumption for a few locations: Bracknell, Valentia and Geneva (Yearly average irradiation can be estimated in these cases with an uncertainty range of 1%).
Systematic significant underestimations are observed in the other data (Albany, Carpentras, Ispra and Trappes) (3 to 8% on a yearly level) whatever the tilt angle. Significant overestimations are only observed at Geneva on the vertical surface during summer months (7%).

DISCUSSION

Several problems may explain the observed discrepancies between measured and computed values. They are discussed below.
The ground albedo is not known exactly at some locations. Furthermore seasonal variations have been observed, for example smaller values at high solar altitude (Ineichen, 1983). The influence of this factor is small in the case of a 45-degree inclination, but it becomes significant for vertical surfaces. A 0.10 absolute error on this factor introduces relative errors in the computation of the irradiation on the surface within 3% (winter) and 10% (summer). This fact probably explains the overestimation of the summer months in Geneva and perhaps a fraction of the discrepancy observed at Ispra.

This problem is irrelevant at the other locations (Trappes, Carpentras and Albany), where there is no ground-reflected radiation (either eliminated by an horizontal black disk or measured separately and subtracted).
The short period of measurements (20 to 75 days for each month in those 3 cases) has to be considered, because R-correlations have been established from 10-year averages. This fact may explain some specific large errors (e.g. August at Trappes, only 33 days of measurement) but surely not the systematic trend. This has been checked in the following way: 10-year averages have been computed at Carpentras from hourly global (horiz. plane) and beam radiation measured values. The hour-by-hour computation method on tilted surfaces is an algorithm by Perez (1986) which has been shown to be the most accurate for this purpose (Bourges, 1986). On a long-term basis the yearly error could be reduced to about -5%, but still remains quite significant (Table 4).
Measurement errors could also be considered in the analysis but cannot be seriously upheld as a general explanation for these 3 locations.
The cause should probably be determined from the correlations themselves, the original data they were based on, and from inaccuracies in Page's algorithm.

CONCLUSION

A set of R-correlations has been presented. It provides a simple computation method for monthly irradiations on tilted planes (facing South) from the corresponding irradiation on a horizontal plane. The comparison with measured data leads to a general accuracy of about 5%, with a tendency to underestimate at some locations. Relative errors are the greatest during winter months and for vertical surfaces (10%). Nevertheless this accuracy may be considered as satisfactory, given the simplicity of the method and the various causes of errors in its application: uncertainties as to the irradiation on a horizontal plane and as to the albedo, and year-to-year variations.

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Table 3 Daily average global irradiation on tilted planes (MJ/m2). Comparison of measured and computed values

	GENEVA			s = 45 deg.		GENEVA			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	5.0	19.5	19.3	10.7	13.4	4.3	12.4	10.0	8.6	8.9
H calc.	5.1	19.4	19.5	10.3	13.2	4.6	12.4	10.6	8.5	9.0
Diff.(%)	1	-1	1	-4	-1	7	0	7	-1	1

	ISPRA			s = 45 deg.		BRACKNELL			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	9.4	17.4	20.0	12.6	14.1	2.9	7.6	7.6	6.8	6.4
H calc.	9.5	16.8	19.7	11.8	13.7	3.1	7.3	7.7	6.7	6.4
Diff.(%)	1	-4	-2	-6	-3	4	-4	1	-2	0

	CARPENTRAS			s = 45 deg.		CARPENTRAS			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	11.8	21.9	23.6	14.7	17.8	11.1	11.9	9.7	11.4	11.2
H calc.	10.5	20.6	22.9	13.7	16.6	9.7	11.1	9.3	10.7	10.1
Diff.(%)	-11	-6	-3	-7	-6	-13	-7	-5	-7	-9

	TRAPPES			s = 45 deg.		TRAPPES			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	5.3	12.8	16.5	9.7	11.8	4.7	7.5	8.1	7.8	7.6
H calc.	4.6	12.2	15.8	8.7	11.0	4.1	7.1	7.7	6.8	7.0
Diff.(%)	-13	-5	-5	-11	-7	-14	-5	-5	-13	-8

	ALBANY			s = 43 deg.		VALENTIA			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	11.1	15.5	19.1	12.9	14.6	4.0	10.6	9.1	7.1	7.4
H calc.	10.7	14.5	18.2	11.6	13.8	3.9	10.5	9.2	6.8	7.3
Diff.(%)	-4	-7	-5	-10	-5	-4	-2	1	-4	-1

Table 4 Idem Table 3. "Measured" values are actually computed using Perez' alg., from hourly measurements (Global hor. + direct) for the period 1972-81

	CARPENTRAS			s = 45 deg.		CARPENTRAS			s = 90 deg.	
Month	Jan.	Apr.	Jul.	Oct.	Year	Jan.	Apr.	Jul.	Oct.	Year
H meas.	11.5	20.6	24.6	16.9	18.2	11.1	12.7	12.4	14.2	12.6
H calc.	10.5	19.6	23.9	15.5	17.2	10.1	12.2	11.9	13.0	11.8
Diff.(%)	-8	-5	-3	-9	-5	-9	-4	-4	-9	-6

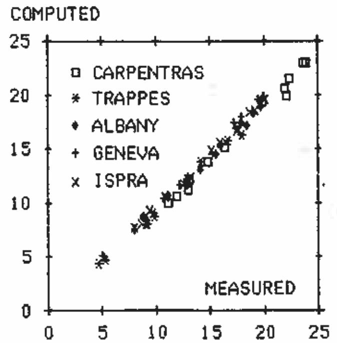

Fig. 2: Measured vs computed values of monthly irradiations (MJ/m2/day) on a tilted plane (slope = 45 deg.) at various locations.